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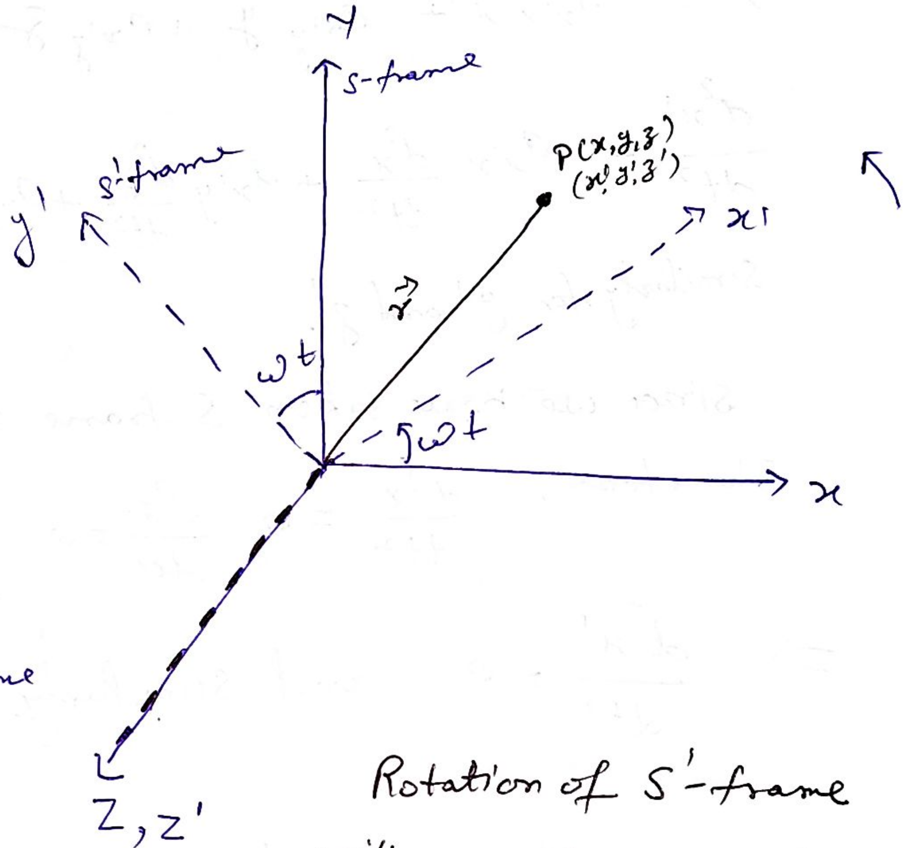
Transformation equations for a rotating frame of reference:



S → An inertial frame

S' → rotating frame

Rotation of S'-frame with a uniform angular velocity ω about the Common Z axis.

In earlier class note, we have derived transformation equations for a frame of reference inclined to an inertial frame. Here we have take S'-frame to be rotating with a angular velocity ω about Z axis which is common for both S and S'-frames.

Here $\vec{r} = x\hat{i} + y\hat{j}$

Following the earlier class note, we can write

$$\left.\begin{aligned} x' &= x\cos\omega t + y\sin\omega t \quad &(a)\\ y' &= -x\sin\omega t + y\cos\omega t \quad &(b)\\ z' &= z \quad &(c)\end{aligned}\right\} \text{——— } (1)$$

The inverse transformation can be written as.

$$x = x'\cos\omega t - y'\sin\omega t \quad \text{——— } 2(a)$$

$$y = x'\sin\omega t + y'\cos\omega t \quad \text{——— } 2(b)$$

$$z = z' \quad \text{——— } 2(c)$$

For S-frame $\frac{d^2x}{dt^2} = 0$, $\frac{d^2y}{dt^2} = 0$ and $\frac{d^2z}{dt^2} = 0$ ——— (3)

Since it is an inertial frame and no force is applied on the particle at point $P(x, y, z)$

Again differentiating Eqns 2(a), 2(b) and 2(c) with respect to t, we obtain

$$\frac{dx'}{dt} = -\omega x\sin\omega t + \omega y\cos\omega t + \frac{dx}{dt}\cos\omega t + \frac{dy}{dt}\sin\omega t$$

using relation 2(b) $\rightarrow y' = -x\sin\omega t + y\cos\omega t$, we write.

$$\frac{dx'}{dt} = +\omega y' + \frac{dx}{dt}\cos\omega t + \cancel{\frac{dx}{dt}\cos\omega t + dy}$$
$$+ \frac{dy}{dt}\sin\omega t \quad \text{——— } (4)$$

Using similar procedure, we obtain

$$\frac{dy'}{dt} = -\omega x\cos\omega t - \omega y\sin\omega t - \frac{dx}{dt}\sin\omega t + \frac{dy}{dt}\cos\omega t$$

Again using $x' = x\cos\omega t + y\sin\omega t$, we obtain.

$$\frac{dy'}{dt} = -\omega x' - \frac{dx}{dt}\sin\omega t + \frac{dy}{dt}\cos\omega t \quad \text{—— (5)}$$

And.

$$\frac{dz'}{dt} = \frac{dz}{dt} \quad \text{—— (6)} \quad \{\text{using Eq. I(c)}\}$$

Next, differentiating Eq. (4) w.r.t. t.

$$\frac{d^2x'}{dt^2} = \omega\frac{dy'}{dt} + \frac{d^2x}{dt^2}\cos\omega t + \frac{d^2y}{dt^2}\sin\omega t - \omega\frac{dx}{dt}\sin\omega t$$
$$+ \omega\frac{dy}{dt}\cos\omega t$$

using Eq (3) we can write above Eq.

$$\frac{d^2x'}{dt^2} = \omega\frac{dy'}{dt} + 0 + 0 - \omega\frac{dx}{dt}\sin\omega t + \omega\frac{dy}{dt}\cos\omega t$$

or $$\frac{d^2x'}{dt^2} = \omega\left[\frac{dy'}{dt} - \frac{dx}{dt}\sin\omega t + \frac{dy}{dt}\cos\omega t\right]$$

using Relation (5) $\frac{dy'}{dt} + \omega x' = -\frac{dx}{dt}\sin\omega t + \frac{dy}{dt}\cos\omega t$

$$\frac{d^2x'}{dt^2} = \omega\left(\frac{dy'}{dt} + \omega x'\right) + \omega\frac{dy'}{dt}$$

or $$\boxed{\frac{d^2x'}{dt^2} = 2\omega\frac{dy'}{dt} + \omega^2 x'} \quad \text{—— (7)}$$

Again we calculate $\frac{d^2y'}{dt^2}$, Using Eq. (5) we can write

$$\frac{d^2y'}{dt^2} = -\omega\frac{dx'}{dt} - \cancel{\frac{d^2x}{dt^2}\sin\omega t}^{\,0} + \cancel{\frac{d^2y}{dt^2}\cos\omega t}^{\,0} - \frac{dx}{dt}\,\omega\cos\omega t - \omega\frac{dy}{dt}\sin\omega t$$

$$\frac{d^2y'}{dt^2} = -\omega\frac{dx'}{dt} - \omega\frac{dx}{dt}\cos\omega t - \omega\frac{dy}{dt}\sin\omega t$$

Using Eq. (4) in above Eq., $\frac{dx'}{dt} - \omega y' = \frac{dx}{dt}\cos\omega t + \frac{dy}{dt}\sin\omega t$

$$\frac{d^2y}{dt^2} = -\omega\frac{dx'}{dt} - \omega\left(\frac{dx'}{dt} - \omega y'\right)$$

or $$\boxed{\frac{d^2y}{dt^2} = -2\omega\frac{dx'}{dt} + \omega^2 y'} \quad \text{—— (8)}$$

From Eq. (6), $$\frac{d^2z'}{dt^2} = \frac{d^2z}{dt^2} \quad \text{—— (9)}$$

From Equations (7) and (8), we see that though there is no force is acting on particle P ~~of~~ in S-frame. but in S'-frame a force seems to be acting on particle. This implies that S'-frame is a non-inertial frame of reference.